

## SUMS OF SMOOTH SQUARES

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ABSTRACT. Let  $R(n, \theta)$  denote the number of representations of the natural number  $n$  as the sum of four squares, each composed only with primes not exceeding  $n^{\theta/2}$ . When  $\theta > e^{-1/3}$  a lower bound for  $R(n, \theta)$  of the expected order of magnitude is established, and when  $\theta > 365/592$ , it is shown that  $R(n, \theta) > 0$  holds for large  $n$ . A similar result is obtained for sums of three squares. An asymptotic formula is obtained for the related problem of representing an integer as the sum of two squares and two squares composed of small primes, as above, for any fixed  $\theta > 0$ . This last result is the key to bound  $R(n, \theta)$  from below.

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