Minisymposium 19 - Random Discrete Structures and Algorithms

Probabilistic Analysis of Local Search Algorithms for TSP

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2-Opt is probably the most basic and widely used local search heuristic for TSP. This heuristic achieves amazingly good results on "real world" instances both with respect to running time and approximation ratio. We present a probabilistic analysis showing that the expected number of improvement steps until 2-Opt terminates on Euclidean instances in which n points are placed uniformly at random in the plane is $\tilde{O}(n^{3+5/6})$ when starting with an initial tour computed by any greedy insertion heuristic. The best previous bound was $\tilde{O}(n^{10})$.
Our probabilistic analysis is not restricted to uniformly random instances. In principle, points can be placed by arbitrary independent continuous distributions with finite support and bounded density. In particular, different points can have different distributions. Our results can be expressed in terms of a *smoothed analysis* in which an adversary selects the initial set of points from $[0,1]^2$ and then these points are randomly perturbed with a Gaussian distribution with standard deviation σ. In this model, we obtain an upper bound of $\tilde{O}(n^{3+5/6}/\sigma)$ on the running time of 2-Opt.
Furthermore, we investigate the behavior of 2-Opt on other input models, e.g., randomly perturbed graphs, and we present an analysis of the approximation achieved by 2-Opt.
Joint work with Matthias Englert and Heiko Roeglin, RWTH Aachen.