Efficient Methods for Aerodynamic Optimization

Ilia Gherman, Volker Schulz

University of Trier, MEGADESIGN-Project

DMV-Jahrestagung
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Outline

- One-shot approach for unconstrained drag minimization and the choice of the reduced Hessian
- Extending the method to include state constraints
- Numerical results
- Conclusions

The MEGADESIGN-Project

- Supported by German Federal Ministry of Economics and Technology

- Main goal of the project : fast algorithms for geometric design of an aircraft

- Partners:
  - German Aerospace Center (DLR)
  - Airbus Germany

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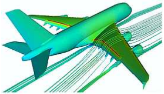
  - University of Trier – Group of Volker Schulz



Research goal in Trier

Question

Is there a fast numerical approach for drag minimization with low relative complexity ?

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Aim

- Construct an optimization algorithm using existing simulation tools
- Overall effort: constant×simulation effort

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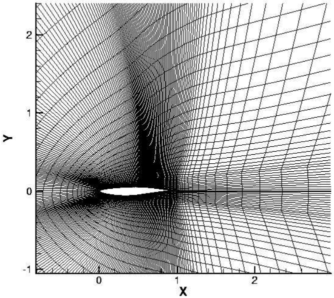
Optimization Problem without Addititional Constraints

$$\min \quad J(u,q)$$
$$\text{subject to} \quad c(u,q) = 0$$



J: the cost function (drag)
c: Euler-flow equations
u: state variable such that the Jacobian C_u is invertible
q: wing profile, parameterized by splines

Black-Box-Methods

Implicit function theorem:

$$\exists\, u : \mathbb{R}^{n_q} \to \mathbb{R}^{n_u}, q \mapsto u(q) \;:\; \forall\, q \;:\; c(u(q), q) = 0.$$

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Reduce the problem

$$\begin{aligned} \min \quad & J(u, q) \\ \text{subject to} \quad & c(u, q) = 0 \end{aligned}$$

to the unconstrained problem

$$\min \quad J(u(q), q) =: I(q)$$

Reduced Gradient via Adjoint Problem

Need to compute the reduced gradient

$$\nabla_q I = -C_q^T C_u^{-T} \nabla_u J + \nabla_q J$$

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sensitivity approach

$$C_q^T C_u^{-T} = \left(C_u^{-1} C_q\right)^T$$

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sensitivity approach

$$C_q^T C_u^{-T} = \left(C_u^{-1} C_q\right)^T$$

adjoint approach

$$C_u^T \lambda = \nabla_u J$$

One-Shot-Approach

Black-box steepest descent method:

- Solve the flow equations (exactly)
- Solve the adjoint equation (exactly) and compute the (exact) reduced gradient based on the adjoint approach
- Update design

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The idea of the one-shot-method

- Based on neccessary optimality conditions
- Solve all three equations simultaneously

One-Shot-Method (1)

Define the Lagrangian functional:

$$\mathcal{L}(u, q, \lambda) = J(u, q) + \lambda^* c(u, q).$$

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Neccessary optimality conditions:

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Use Newton method to solve this system of nonlinear equations!

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One-Shot-Method (2)

Newton-iteration uses KKT-Matrix:

$$\begin{bmatrix} H_{uu} & H_{uq} & C_u^* \\ H_{qu} & H_{qq} & C_q^* \\ C_u & C_q & 0 \end{bmatrix} \begin{pmatrix} \Delta u \\ \Delta q \\ \Delta \lambda \end{pmatrix} = \begin{pmatrix} -\nabla_u \mathcal{L} \\ -\nabla_q \mathcal{L} \\ -c \end{pmatrix}$$

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KKT-Matrix approximated by rSQP-matrix:

$$\begin{bmatrix} 0 & 0 & A^* \\ 0 & \color{red}{B} & C_q^* \\ A & C_q & 0 \end{bmatrix} \begin{pmatrix} \Delta u \\ \Delta q \\ \Delta \lambda \end{pmatrix} = \begin{pmatrix} -\nabla_u \mathcal{L} \\ -\nabla_q \mathcal{L} \\ -c \end{pmatrix}$$

where A is some approximation of C_u.

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Appropriate choice of B

- Exact reduced Hessian

$$B_{\text{ex}} = \begin{bmatrix} -C_u^{-1} C_q \\ I \end{bmatrix}^T \begin{bmatrix} H_{uu} & H_{uq} \\ H_{qu} & H_{qq} \end{bmatrix} \begin{bmatrix} -C_u^{-1} C_q \\ I \end{bmatrix}$$

- “wrong” reduced Hessian

$$B_{\text{inex}} = \begin{bmatrix} -A^{-1} C_q \\ I \end{bmatrix}^T \begin{bmatrix} H_{uu} & H_{uq} \\ H_{qu} & H_{qq} \end{bmatrix} \begin{bmatrix} -A^{-1} C_q \\ I \end{bmatrix}$$

(similar to Bank/Welfert/Yserentant 1990)

- B according to Griewank’s piggy-back concept.

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Theoretical Investigations — Definition of the Quadratic Problem

Quadratic problem (QP):

$$\min_{u,q} \; \frac{1}{2}u^T H_{uu} u + \frac{1}{2}q^T H_{qq} q + f_u^T u + f_q^T q$$
$$\text{subject to} \quad C_u u + C_q q + c = 0.$$

with C_u invertible. Consider the Lagrangian:

$$\mathcal{L}(u, q, \lambda) = \frac{1}{2}u^T H_{uu} u + \frac{1}{2}q^T H_{qq} q + f_u^T u + f_q^T q + \lambda^T (C_u u + C_q q + c).$$

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Theoretical Investigations — Convergence Results

Theorem (Kunisch/Ito/Schulz/Gherman 2006)
There exists an $\eta > 0$, such that the iteration

$$\begin{pmatrix} u^{k+1} \\ q^{k+1} \\ \lambda^{k+1} \end{pmatrix} = \begin{pmatrix} u^{k} \\ q^{k} \\ \lambda^{k} \end{pmatrix} - \begin{bmatrix} 0 & 0 & A^T \\ 0 & B & C_q^T \\ A & C_q & 0 \end{bmatrix}^{-1} \begin{pmatrix} \nabla_u \mathcal{L} \\ \nabla_q \mathcal{L} \\ \nabla_\lambda \mathcal{L} \end{pmatrix}$$

converges to the solution of the (QP), provided

$$\max\{\rho(I - A^{-1} C_u),\ \rho(I - B^{-1} B_{inex})\} < \eta$$

and C_u symmetric.
Proof: Nilpotency of degree 3 of the iteration matrix and perturbation analysis.

Drag Minimization of an RAE 2822 Airfoil

- Flow equations: Euler flow
- Flow-Solver: FLOWer, provided by DLR in forward and adjoint mode (Gauger,. . .)
- Minimize drag (constant profile thickness is preserved in the parameterization of the airfoil)
- Technique employed per iteration:
  - State/Adjoint: single iteration-steps provided by FLOWer
  - Design: rSQP-similar step:

$$\Delta q = -B^{-1} \cdot \gamma_d^k \qquad \text{where } \gamma_d^k = C_q^* (A^*)^{-1} J_u^*,$$

γ_d^k is the current reduced gradient approximation [Hazra/Schulz/Brezillon/Gauger 2005] and B approximates the “wrong” reduced Hessian (BFGS-Updates based on γ_d).

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Results – Unconstrained Optimization

- Fast convergence (total effort < 4 simulations)
- Drastic reduction of the drag . . .

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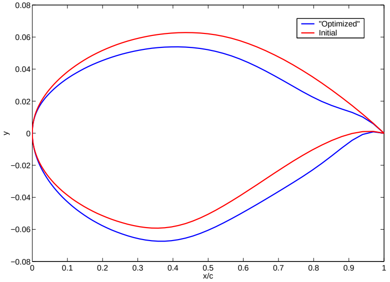

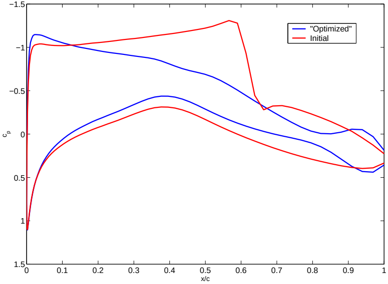

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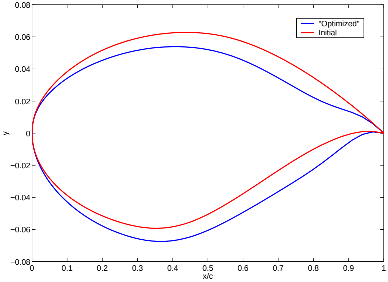

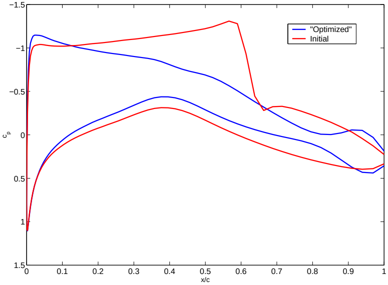

⟶ Neccessary: explicit formulation of aerodynamic constraints

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State Constraints

$$\begin{aligned} \min \quad & J(u, q) \\ \text{subject to} \quad & c(u, q) = 0 \\ & \ell(u, q) \geqslant 0 \qquad \longleftarrow \quad \text{scalar Lift constraint} \end{aligned}$$

Lift depends also on the states and design variables.
The reduced gradient w.r.t lift

$$\gamma_\ell = \frac{d\,\ell}{d\,q}$$

can be computed by the solution of yet another adjoint problem:

$$\gamma_\ell = \nabla_q \ell - C_q^* (A^*)^{-1} \nabla_u \ell$$

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One-Shot with Additonal State Constraints

New Lagrangian:

$$\mathcal{L}(u, q, \lambda, \mu) = J(u, q) + \lambda^* c(u, q) + \mu \ell(u, q).$$

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$\longrightarrow$ partially reduced SQP-method (approximate variant)

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Theoretical Investigations — Additional Constraints

Add to the (QP) the constraint

$$h_u^T u + h_q^T q + h = 0.$$

Convergence of the iteration can be justified analogously to the “unconstrained” theorem. Same conditions with additionally

$$\gamma_\ell = h_q - C_q^T A^{-T} h_u.$$

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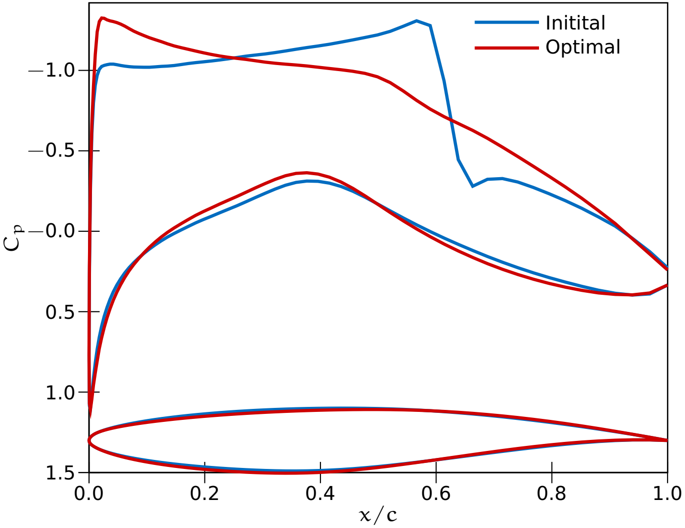
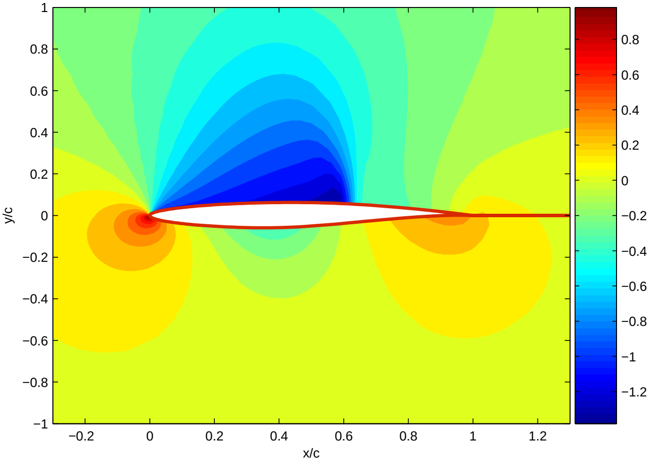
Numerical results (1)

- Minimize drag with constant lift constraint
- Same setting as for the unconstrained optimization
- Reduced gradients w.r.t. drag/lift are computet based on the adjoint solutions after single-iteration steps by `FLOWer`
- Approximations of the reduced Hessian by L-BFGS-updates based on the reduced gradients

Numerical Results (2)



Numerical Results (3)



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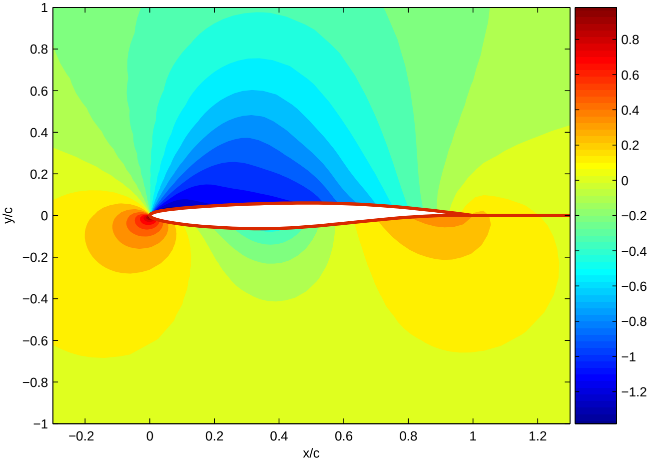
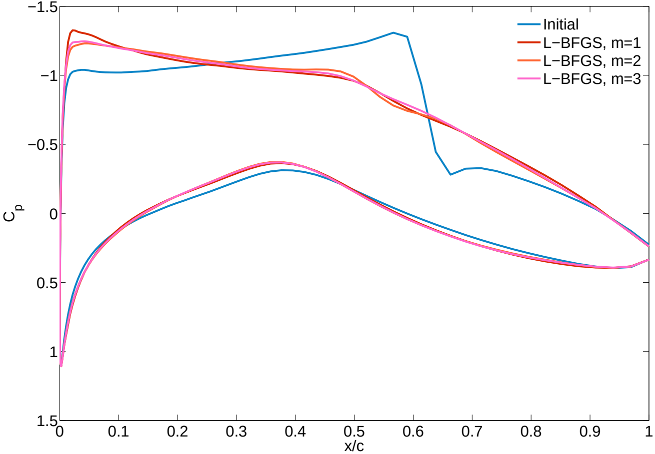


Numerical Results (4)

Pressure Coefficient on the surface of the airfoil



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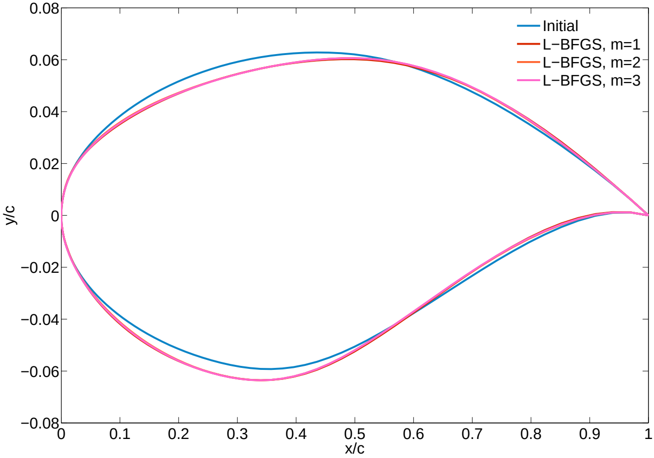
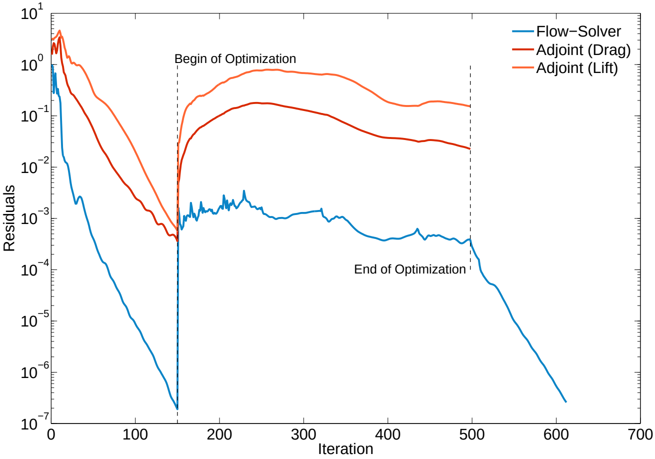
Numerical Results (5)

Airfoils



Numerical Results (6)

L-BFGS-updates with $m = 1$, result: $\frac{\text{optimization effort}}{\text{simulation effort}} < 7$



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Conclusions and Further Research

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- Reduced gradient and Hessian approximations should be consistent with the state/costate solver iteration

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Further Research

- Adding viscosity in the flow equations (Navier-Stokes)
- 3D computations
- Models contain parameters with unknown/uncertain values $\Rightarrow$ robust optimization, stochastic approach

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