Minisymposium 15

Operatortheorie

Leiter des Symposiums:

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Die Operatortheorie beschäftigt sich mit der Analyse linearer Abbildungen auf unendlichdimensionalen Räumen. Einen besonderen Schwerpunkt bildet dabei die Spektraltheorie, die Erweiterungstheorie symmetrischer Operatoren, die Fredholmtheorie und die Theorie der Halbgruppen.

Montag, 18. September

HS IV, Hauptgebäude, Regina-Pacis-Weg

14:30 – 15:20 **Klaus-Jochen Engel** *(University of L'Aquila, Italy)*
Boundary control of flows in networks

15:30 – 15:50 **Andras Bátkai** *(ELTE TTK/Institute of Mathematics)*
Differenzialgleichungen mit Verzögerung in L^p Phasenräumen

16:00 – 16:20 **Peer Kunstmann** *(Karlsruhe)*
L^q-Eigenschaften elliptischer Randwertprobleme

16:30 – 16:50 **Markus Biegert** *(Ulm)*
Elliptic Problems on Varying Domains

Dienstag, 19. September

Hörsaal 411 AVZ I, Endenicher Allee 11-13

15:00 – 15:50 **Christiane Tretter** *(Bremen)*
Spectral problems for block operator matrices in hydrodynamics

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16:00 – 16:20 **Matthias Langer** *(University of Strathclyde, Glasgow)*
Variational principles for eigenvalues of the Klein–Gordon equation

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16:30 – 16:50 **Monika Winklmeier** *(Bremen)*
Estimates for the eigenvalues of the angular part of the Dirac equation in the Kerr-Newman metric

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17:00 – 17:20 **Annemarie Luger** *(TU Berlin)*
On a result for differential operators with singular potentials

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17:30 – 17:50 **Jussi Behrndt** *(University of Groningen)*
Open Quantum Systems

Mittwoch, 20. September

Hörsaal 411 AVZ I, Endenicher Allee 11-13

15:00 – 15:50 **Hagen Neidhardt** *(WIAS Berlin)*
Perturbation theory of semi-groups and evolution equations

16:00 – 16:20 **Bernhard Haak** *(TU Delft)*
A stochastic Datko-Pazy theorem

16:30 – 16:50 **Tanja Eisner** *(Tübingen)*
Fast schwache Konvergenz von Operatorhalbgruppen

17:00 – 17:20 **Carsten Trunk** *(TU Berlin)*
Location of the spectrum of operator matrices which are associated to second order equations

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17:30 – 17:50 **Birgit Jacob** *(TU Delft)*
A resolvent test for admissibility of Volterra observation operators

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Vortragsauszüge

Klaus-Jochen Engel *(University of L'Aquila, Italy)*
Boundary control of flows in networks

We investigate a boundary control problem on a network. We study a transport equation in the network, controlling it in a single vertex. We describe all the possible reachable states and prove a criterium of Kalman type for the vertices in which the problem is controllable.

This is joint work with Marjeta Kramar Fijav (Ljubljana), Rainer Nagel (Tübingen) and Eszter Sikolya (Budapest).

Andras Bátkai *(ELTE TTK/Institute of Mathematics)*
Differenzialgleichungen mit Verzögerung in L^p Phasenräumen

Im Vortrag wird ein halbgruppentheoretischer Zugang zu Differenzialgleichungen mit Verzögerung in L^p Phasenräumen präsentiert. Dies ermöglicht uns die Verzögerung in eine additive Störung zu verwandeln und ermöglicht dadurch die Anwendung der reichen Störungstheorie der Halbgruppen. Neben diesem Zugang werden auch die Ergebnisse neuester spektraltheoretischer Untersuchungen gezeigt.

Referenzen:
Bátkai, A., Piazzera, S., “Semigroups for Delay Equations in L^p-Phase Spaces”, Research Notes in Mathematics vol. 10, A. K. Peters: Wellesley MA, 2005.
Bátkai, A., Eisner, T., Latushkin, Y., *The spectral mapping property of delay semigroups*, submitted

Peer Kunstmann *(Karlsruhe)*
L^q-Eigenschaften elliptischer Randwertprobleme

Wir untersuchen L^q-Eigenschaften elliptischer Randwertprobleme

$$\begin{aligned} \lambda u - Au &= f \quad \text{in } \Omega \subset \mathbb{R}^n \\ Bu &= g \quad \text{auf } \partial\Omega. \end{aligned}$$

Hierbei ist im einfachsten Fall $A = \sum_{jk} a_{jk}\partial_j\partial_k$ ein Differentialoperator mit $a_{jk} \in L^\infty$, $B = \sum_j b_j\partial_j$ ein Differentialoperator erster Ordnung mit $b_j \in C^{0,1}$, sowie $f \in L^q(\Omega)$ und $g \in W^{1,q}(\Omega)$. Ausgehend von Abschätzungen wie

$$|\lambda|\|u\|_q + \|\nabla^2 u\|_q \leq C(\|f\|_q + |\lambda|^{1/2}\|g\|_q + \|\nabla g\|_q)$$

für ein festes $q \in (1,\infty)$ und hinreichend große λ in einem geeigneten Sektor zeigen wir verallgemeinerte Gauß-Abschätzungen und mit deren Hilfe weitere Eigenschaften wie R-Sektorialität, R-Beschränktheit der Lösungsoperatoren und maximale L^p-L^q-Regularität für die induzierte analytische Halbgruppe.

Markus Biegert *(Ulm)*
Elliptic Problems on Varying Domains

The aim of this talk is to show optimal results on local and global uniform convergence of solutions to elliptic equations with Dirichlet boundary conditions on varying domains. We assume that the limit domain be stable in the sense of Keldyš. We further assume that the approaching domains satisfy a necessary condition in the inside of the limit domain, and only require L^2-convergence outside. As a consequence, uniform and L^2-convergence are the same in the trivial case of homogenisation of a perforated domain.

Christiane Tretter *(Bremen)*
Spectral problems for block operator matrices in hydrodynamics

In the linear stability analysis of hydrodynamics, the spectra of non-symmetric systems of coupled differential equations have to be studied. As examples, we consider the Ekman boundary layer problem and the Hagen Poiseuille flow with non-axisymmetric disturbances. In both cases we investigate the essential spectrum by means of operator theoretic methods.

(joint work with M. Marletta, Cardiff)

Matthias Langer *(University of Strathclyde, Glasgow)*
Variational principles for eigenvalues of the Klein–Gordon equation

We consider eigenvalues of the Klein–Gordon equation, which can be written as a quadratic eigenvalue problem. Under certain assumptions the continuous spectrum has a gap and we can characterise eigenvalues in this gap even in the presence of complex eigenvalues. This quadratic eigenvalue problem can also be linearised in a Pontryagin space. Connections between the negative index of the Pontryagin space and the index shift in the variational principle are presented.

Monika Winklmeier *(Bremen)*
Estimates for the eigenvalues of the angular part of the Dirac equation in the Kerr-Newman metric

The radial part of the Dirac equation describing a fermion in the Kerr-Newman background metric has an operator theoretical realisation as a block operator matrix $\mathcal{A} = \left(\begin{smallmatrix} -D & B \\ B^* & D \end{smallmatrix}\right)$ with domain $\mathcal{D}(\mathcal{A}) = \mathcal{D}(B^*) \oplus \mathcal{D}(B)$ in the Hilbert space $\mathcal{H} = L_2(0,\pi)^2$. It can be shown that the spectrum of $\mathcal{A}$ consists of eigenvalues only. We will show that the expression $\mathcal{A} - \lambda$ allows for a factorisation into three factors such that all the information about the spectrum of $\mathcal{A}$ is contained in a scalar operator valued function. From this function we obtain a lower bound for the smallest eigenvalue in modulus of $\mathcal{A}$. Another method to obtain such a bound is to use techniques related to the quadratic nuermical range of block operator matrices.

Annemarie Luger *(TU Berlin)*
On a result for differential operators with singular potentials

We explore the connection between a (generalized) Titchmarsh-Weyl-coefficient for the singular Sturm-Liouville operator

$$\ell(y) := -y''(x) + \left(\frac{q_0}{x^2} + \frac{q_1}{x}\right) y(x) \quad \text{on } x \in (0,\infty),$$

with $q_0 > \frac{3}{4}$ and $q_1 \in \mathbb{R}$, and a certain singular perturbation of this operator.

This talk is based on joint work with Pavel Kurasov (Lund).

Jussi Behrndt *(University of Groningen)*
Open Quantum Systems

Open quantum systems are often described with a maximal dissipative operator A_D, a so-called pseudo-Hamiltonian, and a self-adjoint operator A_0 in some Hilbert space $\mathcal{H}$. If L denotes a minimal self-adjoint dilation of A_D, i.e., L acts in a Hilbert space $\mathcal{H} \oplus L^2(\mathbb{R}, \mathfrak{K})$ such that $P_{\mathcal{H}}(L - \lambda)^{-1}|_{\mathcal{H}} = (A_D - \lambda)^{-1}$, and $L_0 = A_0 \oplus -i\frac{d}{dx}$, then the scattering matrix of the closed system $\{L, L_0\}$ can be recovered from the scattering matrix of the dissipative system $\{A_D, A_0\}$. Since in this model L is not semibounded from below serious doubts arise from a physical point of view.
We propose a slightly different approach where instead of a fixed pseudo-Hamiltonian A_D a family of energy dependent pseudo-Hamiltonians $\{A_{-\tau(\lambda)}\}$ is considered. The outer space $L^2(\mathbb{R}, \mathfrak{K})$ is replaced by some Hilbert space $\mathcal{K}$ and the Hamiltonian L in $\mathcal{H} \oplus \mathcal{K}$ satisfies $P(L - \lambda)^{-1}|_{\mathcal{H}} = (A_{-\tau(\lambda)} - \lambda)^{-1}$ and is often semibounded from below. We show that the scattering matrix of the closed system can be recovered in a similar way as above and that the model with one fixed pseudo-Hamiltonian can be regarded as an approximation. The abstract theory is illustrated with some examples.

The talk is based on joint work with Mark M. Malamud (Donetsk National University, Ukraine) and Hagen Neidhardt (WIAS, Berlin).

Hagen Neidhardt *(WIAS Berlin)*
Perturbation theory of semi-groups and evolution equations

The aim of the present talk is to develop an approach to the Cauchy problem for linear evolution equations of type

$$\frac{\partial}{\partial t}u(t) + A(t)u(t) = 0, \quad u(s) = u_s, \quad a < s \leq t < b,$$

on a separable Banach space X, where (a, b) is a finite open interval and $\{A(t)\}_{t\in(a,b)}$ is a family of closed linear operators on the separable Banach space X. The main question concerning the Cauchy problem is to find a so-called “solution operator” or propagator $U(t, s)$. We are going to solve this problem embedding it into a perturbation problem for generators of semi-groups in the Banach space $L^p([0, T], X)$, $1 < p < \infty$. The abstract existence results are applied to Schrödinger operators with time-dependent point interactions.

Bernhard Haak *(TU Delft)*
A stochastic Datko-Pazy theorem

The well-known Datko-Pazy theorem states that if $(T(t))_{t\geq 0}$ is a strongly continuous semigroup on a Banach space E such that all orbits $T(\cdot)x$ belong to the space $L^p(\mathbb{R}_+, E)$ for some $p \in [1, \infty)$, then $(T(t))_{t\geq 0}$ is uniformly exponentially stable, or equivalently, there exists an $\epsilon > 0$ such that all orbits $t \mapsto e^{\epsilon t}T(t)x$ belong to $L^p(\mathbb{R}_+, E)$. We show that a similar result also hold for so-called γ–radonifying operators, namely the equivalence of

1. For all $x \in E$, $T(\cdot)x \in \gamma(\mathbb{R}_+, E)$.
2. There exists an $\epsilon > 0$ such that for all $x \in E$, $t \mapsto e^{\epsilon t}T(t)x \in \gamma(\mathbb{R}_+, E)$.

If E is a Hilbert space, $\gamma(\mathbb{R}_+, E) = L^2(\mathbb{R}_+, E)$ and we reobtain Datko's theorem mentioned above. γ–radonifying operators play an important role in the study abstract stochastic Cauchy problems on E whence the result can also be seen as a perturbation result for stochastic Cauchy problems.

References:

B. Haak, M. Veraar, J. van Neerven: *A stochastic Datko-Pazy theorem*, submitted; avaiable on ArXiv.

Tanja Eisner *(Tübingen)*
Fast schwache Konvergenz von Operatorhalbgruppen

Für C_0-Halbruppen auf Banachräumen diskutieren wir den Zusammenhang zwischen Spektraleigenschaften des Generators und der Konvergenz der Halbgruppe für $t \to \infty$ (insbesondere für die schwache Topologie).

Carsten Trunk *(TU Berlin)*
Location of the spectrum of operator matrices which are associated to second order equations

We study second order equations of the form

$$\ddot{z}(t) + A_o z(t) + D\dot{z}(t) = 0.$$

Here the stiffness operator A_o is a possibly unbounded positive operator on a Hilbert space H, which is assumed to be boundedly invertible, and D, the damping operator, is an unbounded operator, such that $A_o^{-1/2} D A_o^{-1/2}$ is a bounded non negative operator on H. This second order equation is equivalent to the standard first-order equation $\dot{x}(t) = Ax(t)$, where $A : \mathcal{D}(A) \subset \mathcal{D}(A_o^{1/2}) \times H \to \mathcal{D}(A_o^{1/2}) \times H$, is given by

$$A = \begin{bmatrix} 0 & I \\ -A_o & -D \end{bmatrix},$$

$$\mathcal{D}(A) = \left\{ \left[\begin{smallmatrix} z \\ w \end{smallmatrix} \right] \in \mathcal{D}(A_o^{1/2}) \times \mathcal{D}(A_o^{1/2}) \mid A_o z + Dw \in H \right\}.$$

This block operator matrix has been studied in the literature for more than 20 years.
It is well-known that A generates a C_0-semigroup of contraction, and thus the spectrum of A is located in the closed left half plane.
We are interested in a more detailed study of the location of the spectrum of A in the left half plane. In general the (essential) spectrum of A can be quite arbitrary in the closed left half plane. Under various conditions on the damping operator D we describe the location of the spectrum and the essential spectrum of A.

The talk is based on joint work with Birgit Jacob (Delft).

Birgit Jacob *(TU Delft)*
A resolvent test for admissibility of Volterra observation operators

Necessary and sufficient conditions are given for finite-time admissibility of a linear system defined by a Volterra integral equation when the underlying semigroup is equivalent to a contraction semigroup. These necessray and sufficient conditions are in terms of a pointwise bound on the resolvent of the infinitesimal generator. This generalizes an analogous result known to hold for the standard Cauchy problem.

The talk is based on joint work with Jonathan R. Partington (University of Leeds).