Exercises in Geometry II

University of Bonn, Summer Semester 2018
Dozent: PD Dr. Fernando Galaz-Garcia
Assistant: Saskia Roos
Sheet 1

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1. Induced metrics [4 points]

Let (M, g) be a complete Riemannian manifold and $N \subset M$ be a closed embedded submanifold, i.e. N is compact without boundary.
Show that g induces a complete metric on N.

2. Integral curves and geodesics [4 points]

Let (M, g) be a Riemannian manifold and $f : M \to \mathbb{R}$ be a smooth function on M with the property $|\operatorname{grad} f| \equiv 1$.
Show that the integral curves of $\operatorname{grad} f$ are geodesics.

3. Riemannian covering [4 points]

Let $p : \tilde{M} \to M$ be a smooth covering of a Riemannian manifold (M, g).
a) Show that there is a metric $\tilde{g}$ on $\tilde{M}$ such that $p : (\tilde{M}, \tilde{g}) \to (M, g)$ is a local isometry.
b) Show that $(\tilde{M}, \tilde{g})$ is complete if and only if (M, g) is complete.

4. Non-complete Riemannian manifolds [4 points]

Give an example of a non-complete connected Riemannian manifold (M, g) such that for any two point p and g can be joined by a distance realizing geodesic in M.

Due on Monday, April 30

Homepage of the lecture: `https://www.math.uni-bonn.de/people/galazg/`