Part I: Platonic Solids*

Relations among them, Simple Truncations
Part II below: **Archimedean Solids,**
includes: Cubeoctahedron, Icosidodecahedron, Buckyball
Part III below: **Dual Polyhedra**

There are five (and only five) Platonic solids. Three of them are easy to imagine — Cube, Octahedron and Tetrahedron — while the remaining two are more difficult: Icosahedron and Dodecahedron. The earliest known models date from the Stone Age.

What to do in 3D-XplorMath?

First, the program shows how the other four Platonic solids are obtained from the Cube: Select first one of the other polyhedra, then in the Action Menu:

`Show Relation with Cube`.

For the Octahedron one sees that its six vertices are the midpoints of the faces of a cube; the Octahedron faces are equilateral triangles. The Tetrahedron sits in the cube so that its four vertices are vertices of the Cube, and the six Tetrahedron edges are face diagonals of the Cube. The Icosahedron can be placed inside a Cube so that its twelve vertices lie on the six faces of the Cube: see the default

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* This file is from the 3D-XplorMath project. Please see:
http://3D-XplorMath.org/

morph in 3DXM, preferably when `Show Relation with Cube` is chosen. The Dodecahedron can be placed around a Cube so that its twelve pentagon faces rest on the twelve edges of the cube. See again the default morph going from the Rhombic Dodecahedron via the Platonic Dodecahedron to a cube with subdivided faces.

Second, by cutting off appropriately the vertices or the edges of a Platonic solid one obtains the simpler ones of the **Archimedean solids**. These are polyhedra whose faces are (several kinds of) regular polygons, whose edges all have the same length and whose vertices all look the same. Select in the Action Menu any of the three `Truncations` and then do the associated default Morph. For example, the `Edge Truncations` morph from one Platonic solid to another one (which is called its dual).

Third, since our intuition handles two dimensions better than three, it is interesting to project the Platonic solids from their midpoints onto a circumscribed sphere. View in Wire Frame and select in the Action Menu:

`Show Central Projection to Sphere` .

These two-dimensional spherical views of the Platonic solids come very close to explaining why there are no other such beautiful polyhedra.

Fourth, the Icosahedron and the Dodecahedron have very beautiful Stellations, polyhedra that fascinated Kepler. Select (Action Menu): `Create Stellated`. Note that all the mentioned views have their own default morphs. Kepler imagined the stellated Dodecahedron as having pentagon stars as faces. One can also select it in the Polyhedra Menu as `Kepler's Great Dodecahedron`. In 3DXM it is drawn as a (negative) stellation of the Icosahedron.

Fifth, for the Cube and the Icosahedron there are two special entries in the Action Menu when viewing these solids in Wire Frame. For the Cube select `Show Intersection With Plane`, preferably in one of the stereo modes. The plane is represented by random dots and the dots inside the Cube are deleted; the Cube can be rotated and moved forward and backward, always showing its polygonal intersection with the plane. For the Icosahedron select in the Action Menu `Add Borromean Link`, preferably in one of the stereo modes. Note how the boundaries of the emphasized rectangles are intertwined or linked. The edge lengths of each rectangle are equal to the lengths of an edge and a diagonal of a regular pentagon, thus showing the relation of the Icosahedron inside the Cube with the Golden Ratio. Also, the default morph of this image is worth viewing.

Finally, stone objects with Platonic Symmetry were found, mainly in Scotland. They were dated 2500 B.C. They are carved from bigger stones, but they look as if they were conceived as collections of balls. Therefore we have added the Action Menu entry: `Show As Stone Balls`. In Patch Display the balls are fine triangulations of the Bucky Ball, in Wire Frame the balls are shown with random dots. One can also view other sphere triangulations after one has selected (in Patch Display) `Create Subdivided`: another entry appears: `Triangulate Further`.

Part II: Archimedean Solids

Here is the definition again: All faces are regular polygons (of up to three different kinds). All edges have the same length. All vertices (with their outgoing edges) are congruent. In addition to the five Platonic solids there are twelve of them.

We have already seen the simplest ones: Truncate the vertices of a Platonic solid; there are two possibilities, if some portion of the edges remain this is called `Standard Truncation`, and if the truncation cuts go through the midpoints of the edges we have a `Midpoint Truncation`. One can also `truncate the edges`; this deformation leads to the same Archimedean solid if one starts from the Octahedron or the Cube, and also if one starts from the Icosahedron or the Dodecahedron. Two more are obtained if one truncates the edges and the vertices; the deformation is easier to observe if one uses the standard truncation on either the Cubeoctahedron or the Icosidodecahedron. In fact, not quite the standard truncation, because that would make rectangles instead of squares from the truncated vertices.

Finally there are the `Snub Polyhedra`. We could not find what 'snub' means in this context. We describe the construction and call it 'to snub'. Each face of a Platonic solid is scaled down from its midpoint and also rotated around the midpoint. The 'snubbed' polyhedron is the convex hull of these deformed faces. This 2-parameter deformation can be adjusted to give a 1-parameter family of polyhedra whose faces are either regular polygons or isosceles triangles. In each family is an Archimedean solid. A snubbed Tetrahedron is an Icosahedron, snubbed Cube and Octahedron give the same Archimedean solid and also snubbed Dodecahedron and Icosahedron agree.

One can probably understand all these truncations better if one selects in the Action Menu

`Snub Or Truncate Polyhedron In Polyhedron.`

This will add the original polyhedron (as wire frame) to the truncation.

Adapted Morphs: Each selection in the Action Menu will cause that the default deformation, `Morph` in the Animation Menu, is adjusted to the Action Menu selection.

Part III: Dual Polyhedra

Duality has the simplest definition for Platonic solids: The convex hull of the centers of the faces is the dual polyhedron. The following definition works for Platonic and Archimedean solids: Take the tangent planes to the circumscribed sphere at the vertices of the polyhedron; consider them as the boundaries of halfspaces which contain the polyhedron; the intersection of these halfspaces is the dual polyhedron. For example the rhombic dodecahedron is dual to the cubeoctahedron. Similarly, consider convex

polyhedra whose inscribed sphere touches **all** faces; the convex hull of the contact points of the faces with the inscribed sphere is the dual polyhedron.
These definitions of duality are unique up to scaling. For a general convex polyhedron definitions are no longer unique up to scaling. Choose an interior point and project the polyhedron radially onto a sphere around the chosen point; one obtains a tessalation of the sphere with the same combinatorial properties as the boundary of the polyhedron; consider on the sphere the Dirichlet or Voronoi domains of the vertices (the points which are closer to one vertex than to all others); these domains form the dual tessalation; any polyhedron whose radial projection to the sphere has the same combinatorics as this dual tessalation, is called a dual polyhedron to the given one.

In 3D-Xplormath one can view all Archimedean solids inside their dual polyhedra: First select one of the Platonic solids from the Polyhedra Menu. Then check in the Action Menu the entry:

`Show All Polyhedra Inside Their Duals` .

Now, each of the provided truncations gives an Archimedean solid, shown inside its dual polyhedron. These images can be morphed, including the surrounding dual polyhedra.

H.K.