Tractrix *

The Tractrix is a curve with the following nice interpretation: Suppose a dog-owner takes his pet along as he goes for a walk "down" the y-axis. He starts from the origin, with his dog initially standing on the x-axis at a distance aa away from the owner. Then the Tractrix is the path followed by the dog if he "follows his owner unwillingly", i.e., if he constantly pulls against the leash, keeping it tight. This means mathematically that the leash is always tangent to the path of the dog, so that the length of the tangent segment from the Tractrix to the y-axis has constant length aa. Parametric equations for the Tractrix (take $bb = 0$) are:

$$x(t) = aa \cdot \sin(t)(1 + bb)$$
$$y(t) = aa \cdot (\cos(t)(1 + bb) + \ln(\tan(t/2))).$$

The curves obtained for $bb \neq 0$ are generated by the same kinematic motion, except that a different point of the moving plane is taken as the drawing pen. See the default `Morph`.

The Tractrix has a well-known surface of revolution,

* This file is from the 3D-XplorMath project. Please see: http://3D-XplorMath.org/

called the Pseudosphere, Namely, rotating it around the y-axis gives a surface with Gaussian curvature -1. This means that the Pseudosphere can be considered as a portion of the Hyperbolic Plane. The latter is a geometry that was discovered in the 19th century by Bolyai and Lobachevsky. It satisfies all the axioms of Euclidean Geometry except the Axiom of Parallels. In fact, through a point outside a given line (= geodesic) there are infinitely many lines that are parallel to (i.e., do not meet) the given line.

There are many connections, sometimes unexpected, between planar curves. For the Tractrix select: `Show Osculating Circles And Normals`. One observes a Catenary (see another entry in the curve menu) as the envelope of the normals.

H.K.