V5A1: ADVANCED TOPICS IN ALGEBRA: ÉTALE COHOMOLOGY

The goal of this course will be to teach étale cohomology, following [1] primarily, with additional references below. Étale cohomology was developed by Grothendieck in 1960 with the goal of proving the Weil conjectures.

The Weil conjectures are a series of statements about the generating functions obtained by counting the number of points of smooth projective varieties over finite fields; these generating functions are called zeta functions by analogy with the Riemann zeta function. We will introduce zeta functions of algebraic varieties and the Weil conjectures in the beginning of the course as motivation.

The étale cohomology groups of algebraic varieties or schemes are analogues of singular cohomology groups of topological spaces. For example, if $X/\mathbb{C}$ is an algebraic variety and $\mathbb{F}_q$ is a finite field then $H^i_{\text{et}}(X, \mathbb{F}_q)$ agrees with the singular cohomology of the underlying complex manifold $H^i(X(\mathbb{C}), \mathbb{F}_q)$. Étale cohomology gives a substitute for singular cohomology in the case of varieties defined over finite fields and has many of the nice properties of singular cohomology. On the other hand, if $X = \text{Spec } K$ for a field K, then the étale cohomology of X agrees with the Galois cohomology of K. More generally, étale cohomology interpolates between the first setting (an algebraic variety $X/\mathbb{C}$), which is purely geometric, and the second one ($X = \text{Spec } K$), which is purely arithmetic.

Prerequisites

The prerequisites for the course are two first courses in algebraic geometry, for example covering the material in [3]. Some background in commutative algebra and category theory could also be helpful.

Syllabus

We will cover topics such as: faithfully flat descent, Grothendieck topologies, étale morphisms, cohomology of curves, proper and smooth base change, the Lefschetz fixed-point formula.

References

[1] P. Deligne, *Cohomologie etale*, SGA 4 1/2, Lecture Notes in Math 569, Springer-Verlag.
[2] E. Freitag, R. Kiehl, *Etale cohomology and the Weil conjectures*, Springer-Verlag.
[3] R. Hartshorne, *Algebraic geometry*, Graduate texts in mathematics 52, Springer-Verlag.
[4] J. Milne, *Etale cohomology*, Princeton University Press.
[5] G. Tamme, *Introduction to etale cohomology*, Springer Universitext.